WS#7-3
The Law of Cosines

1. **The Law of Cosines states** For a triangle w/ sides a, b, c and opposite angles α, β, γ, respectively: $c^2 = a^2 + b^2 - 2ab\cos\gamma$

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

2. **Possibilities:**

Case 1: SAS

Solve the triangle: $a = 2,\ b = 3,\ \gamma = 60°$

① $c = \sqrt{(2)^2 + (3)^2 - 2(2)(3)\cos 60°}$

$c = \sqrt{7}$

② $\alpha = \cos^{-1}\left[\frac{b^2 + c^2 - a^2}{2bc}\right] = \cos^{-1}\left[\frac{(3)^2 + (\sqrt{7})^2 - (2)^2}{2(3)(\sqrt{7})}\right]$

$\alpha \approx 40.9°$

③ $\beta = 180 - (60 + 40.9) = 79.1°$

Case 2: SSS

Solve the triangle: $a = 4,\ b = 3,\ c = 6$

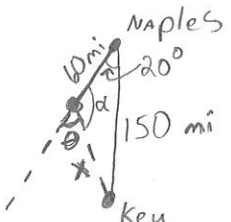
① $\alpha = \cos^{-1}\left[\frac{b^2 + c^2 - a^2}{2bc}\right] = \cos^{-1}\left[\frac{(3)^2 + (6)^2 - (4)^2}{2(3)(6)}\right] \approx 36.3°$

② $\beta = \cos^{-1}\left[\frac{a^2 + c^2 - b^2}{2ac}\right] = \cos^{-1}\left[\frac{(4)^2 + (6)^2 - (3)^2}{2(4)(6)}\right] \approx 26.4°$

③ $\gamma = 180 - (36.3 + 26.4)$

$\gamma = 117.3°$

3. A motorized sailboat leaves Naples, Florida, bound for Key West, 150 miles away. Maintaining a constant speed of 15mph, but encountering heavy crosswinds and strong currents, the crew finds, after 4 hours, that the sailboat is off course by 20°.
   A. How far is the sailboat from Key West at this time?
   B. Through what angle should the sailboat turn to correct its course?
   C. How much time has been added t the trip because of this? (Assume that the speed remains at 15mph.)



A.) $x = \sqrt{(60)^2 + (150)^2 - 2(60)(150)\cos 20°}$

$x = 95.8$ mi (Distance from the sailboat to Key West)

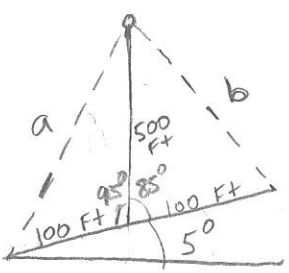
B.) $\alpha = \cos^{-1}\left[\frac{(60)^2 + (95.8)^2 - (150)^2}{2(60)(95.8)}\right] = 147.2°$

$\theta = 180 - \alpha = 180 - 147.2 = 32.8°$

C.) Total trip length is now $60 + 96 = 156$ mi

* The extra 6 miles will require Abt $\frac{6}{15} = .4$ hour or 24 minutes if the speed of 15 mi/hr is maintained

4. A radio tower 500 feet high is located on the side of a hill with an inclination to the horizontal of 5°. How long should two guy wires be if they are to connect to the top of the tower and be secured at two points 100 feet directly above and below the base of the tower?



$b = \sqrt{(100)^2 + (500)^2 - 2(100)(500)\cos 85°}$

$b = 501.28$ Ft

$a = \sqrt{(100)^2 + (500)^2 - 2(100)(500)\cos 95°}$

$a = 518.38$ Ft